LINE, TRIANGLES & OTHER POLYGONS

Point, line and plane

There are three basic concepts in geometry. These concepts are the “point”, “line” and “plane”.

Point

A fine dot, made by a sharp pencil on a sheet of paper, resembles a geometrical point very closely. The sharper the pencil, the closer is the dot to the concept of a geometrical point.

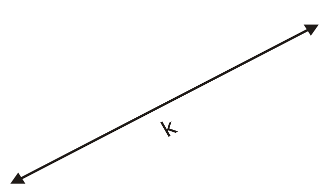
Plane

The surface of a smooth wall or the surface of a sheet of paper or the surface of a smooth blackboard are close examples of a plane. The surface of a blackboard is, however, limited in extent and so are the surfaces of a wall and a sheet of paper but the geometrical plane extends endlessly in all directions.

Lines

A line in geometry is always assumed to be a straight line. It extends infinitely far in both directions. It is determined if two points are known. It can be expressed in terms of the two points, which are written in capital letters. The following line is called AB.

Or, a line may be given one name with a small letter. The following line is named line **k**.



Notes / Rough Work

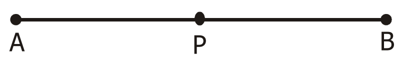
POINT TO REMEMBER

We can draw infinitely many lines from a single point.

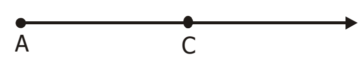
A line segment is a part of a line between two endpoints. It is named by its endpoints, for example, A and B.

AB is a line segment. It has a definite length.

If point P is on the line and at the same distance from A as well as from B, then P is the **midpoint** of segment AB. When we say AP = PB, we mean that the two line segments have the same length.

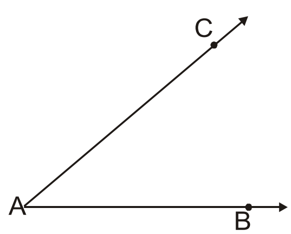


A part of a line with one endpoint is called a ray. AC is a ray of which A is an endpoint. The ray extends infinitely far in the direction away from the endpoint.



Angles

An angle is formed by two rays with the same initial point. So an angle looks like the one in the figure below.



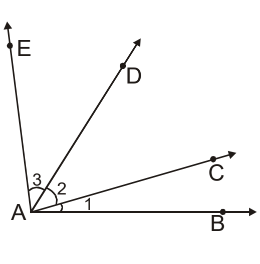
Definition : An angle is the union of two non-collinear rays with a common initial point.

The two rays forming an angle are called the "arms" of the angle and the common initial point is called the "vertex" of the angle.

Notation : The angle formed by the two rays AB and AC, is denoted by the symbol $\angle BAC$ or $\angle CAB$.

It is at times convenient to refer the angle BAC, simply as $\angle A$. However this cannot be done if there are more than one angle, with the same vertex A.

Also, it is convenient to denote angles by symbols such as $\angle 1$, $\angle 2$, $\angle 3$, etc.



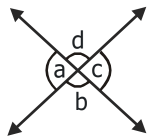
Notes / Rough Work

Types of angles

Notes / Rough Work

Vertical angles

When two lines intersect, four angles are formed. The angles opposite to each other are called **vertical** angles and are equal to each other.

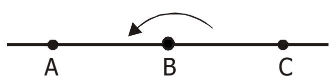


a and c are vertical angles, ∠ a = ∠ c.

b and d are vertical angles, ∠ b = ∠ d.

Straight Angle

A straight angle has its sides lying along a straight line. It is always equal to 180°.

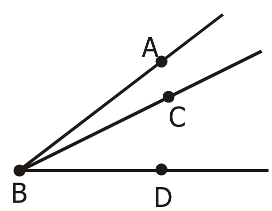


∠ ABC = ∠ B = 180°.

i.e., ∠ B is a straight angle.

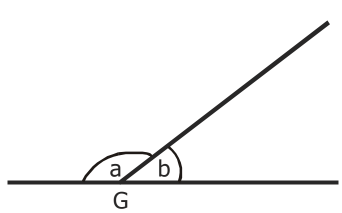
Adjacent angles

Two angles are **adjacent** if they share the same vertex and a common side but no angle is inside the other angle. ∠ABC and ∠CBD are adjacent angles. Even though they share a common vertex B and a common side AB, ∠ ABD and ∠ ABC are not adjacent angles because one angle is inside the other.



Supplementary angles

If the sum of two angles is a straight angle (180°), the angles are **supplementary** and each angle is the supplement of the other.



∠ G is a straight angle = 180°

∴ ∠ a + ∠ b = 180°. ⇒ ∠ a and ∠ b are supplementary angles.

E1. What is the supplement of an angle whose measure is 57°?

Sol. The supplement = 180° – 57° = 123°.

E2. The supplement of an angle is one third of the angle. Find the supplement of the angle.

Sol. Let the angle be x. So, the supplement is x/3.

Now, x + x/3 = 180° or 4x/3 = 180° or x = 135°.

Hence, the supplement = 135/3 = 45°.

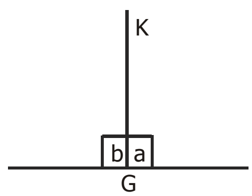
Right angles

If two supplementary angles are equal, they are both **right** angles. A **right** angle is one-half of 180°. Its measure is 90°. A right angle is symbolised by ∟ .

∠G is a straight angle.

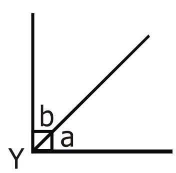
∠b + ∠a = ∠G, and ∠a = ∠b.

∴ ∠a and ∠b are right angles.



Complementary angles

If the sum of two angles is 90°, the angles are **complementary** and each angle is the complement of the other.



For example, if ∠ Y is a right angle and

∠ a + ∠ b = ∠ Y = 90°, then

∠ a and ∠ b are complementary angles.

E3. What is the complement of an angle whose measure is 47°?

Sol. The complement = 90° – 47° = 43°.

E4. The complement of an angle is two third of the angle. Find the complement of the angle.

Sol. Let the angle be x. So, the complement is 2x/3.

Now, x + 2x/3 = 90° or 5x/3 = 90° or x = 54°.

Hence, the complement = $\frac{54 \times 2}{3} = 36°$.

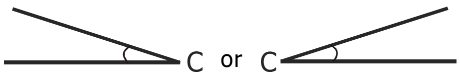
Notes / Rough Work

POINT TO REMEMBER

Two parallel lines never meet, and if two lines are not parallel they will definitely have a point of intersection.

Acute angles

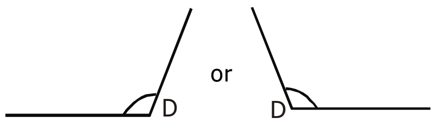
Acute angles are the angles whose measure is less than 90°. No two acute angles can be supplementary. Two acute angles can be complementary angles.



∠ C is an acute angle.

Obtuse angles

Obtuse angles are the angles whose measure is greater than 90° and less than 180°.

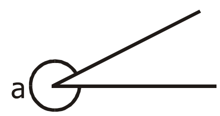


∠ D is an obtuse angle.

Reflex angle

An angle with measure more than 180° and less than 360° is called a reflex angle.

$180° < a < 360°$



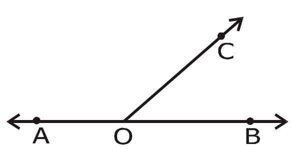
E5. Categorise the following angles as acute, right, obtuse, reflex and straight angle : 37°, 49°, 138°, 90°, 180°, 89°, 91°, 112°, 45°, 235°.

Sol. Acute angles : 37°, 49°, 45°, 89°.
Right angle : 90°.
Obtuse angles : 138°, 91°, 112°.
Reflex angle : 235°.
Straight angle : 180°.

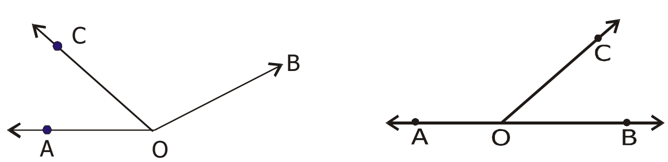
Some properties of angles

Linear pair of angles

From the figure ∠ AOC and ∠ BOC are adjacent angles. Look at the non-common arms, OA and OB. These are two opposite rays or these are collinear. We have a name for them also.



Definition : Two adjacent angles are said to form a linear pair of angles if their non-common arms are two opposite rays. Now, look at the figure below



(i) (ii)

Notes / Rough Work

POINT TO REMEMBER

One and only one line can pass through two distinct points.

In each of the figures above, ∠ AOC and ∠ COB form a pair of adjacent angles. In figure (ii) we have a linear pair of angles. If adjacent angles ∠ AOC and ∠ COB form a linear pair, then

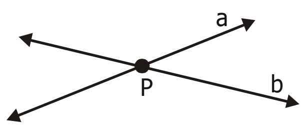
∠ AOC + ∠ COB = 180°.

Imp.

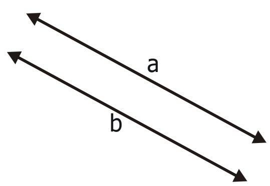
Two adjacent angles are a linear pair of angles if and only if they are supplementary.

Parallel lines

Two lines meet or intersect if there is one point that is on both lines. Two different lines may either intersect at a single point or never intersect, but they can never meet in more than one point.

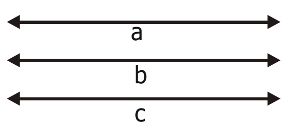


Two lines in the same plane that never meet no matter how far they are extended are said to be parallel, for which the symbol is ||. In the following diagram a || b.

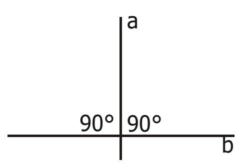


Imp.

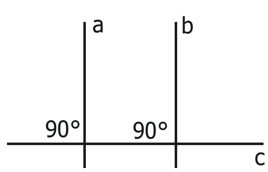
- If two lines in the same plane are parallel to a third line, then they are parallel to each other. Since a || b and b || c, we know that a || c.



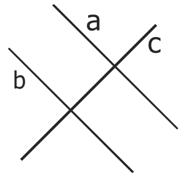
- Two lines that meet each other at right angle are said to be perpendicular, for which the symbol is ⊥. Line a is perpendicular to line b.



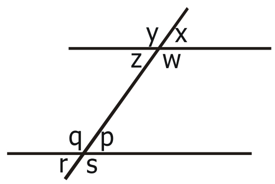
- Two lines in the same plane that are perpendicular to the same line are parallel to each other.



- Line ‘a’ ⊥ line ‘c’ and line ‘b’ ⊥ line ‘c’. ∴ a || b. A line intersecting two other lines is called a **transversal**. Line ‘c’ is a transversal intersecting lines ‘a’ and ‘b’.



The four angles between the given lines are called **interior angles** and the four angles outside the given lines are called **exterior angles.** If two angles are on opposite sides of the transversal, they are called **alternate angles.**



∠z, ∠w, ∠q and ∠p are interior angles.

∠y, ∠x, ∠r and ∠s are exterior angles.

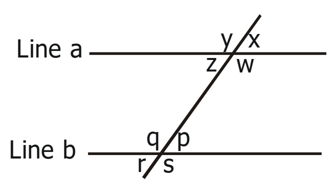
∠z and ∠p are alternate interior angles, so are ∠w and ∠q.

∠y and ∠s are alternate exterior angles, so are ∠x and ∠r.

Pairs of corresponding angles are ∠y and ∠q; ∠z and ∠r; ∠x and ∠p, and ∠w and ∠s.

Two lines intersected by a transversal are parallel, if

1. the corresponding angles are equal, or
2. the alternate interior angles are equal, or
3. the alternate exterior angles are equal, or
4. interior angles on the same side of the transversal are supplementary.



If a || b, then

1. ∠y = ∠q. ∠z = ∠r, ∠x = ∠p and ∠w = ∠s.
2. ∠z = ∠p and ∠w = ∠q.
3. ∠y = ∠s and ∠x = ∠r.
4. ∠z + ∠q = 180° and ∠p + ∠w = 180°.

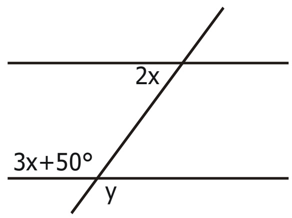
Since vertical angles are equal, ∴ ∠p = ∠r, ∠q = ∠s, ∠y = ∠w, and ∠x = ∠z. If any one of the four conditions for equality of angles holds true, then the lines are parallel.

Notes / Rough Work

POINT TO REMEMBER

Vertically opposite angles are congruent.

E6. In the given figure, two parallel lines are intersected by a transversal. Find the measure of angle y.



Sol. The two labelled angles are supplementary.

$\therefore$ 2x + (3x + 50°) = 180°.

5x = 130° $\Rightarrow$ x = 26°.

Since $\angle$y is vertical to the angle whose measurement is 3x + 50°, it has the same measurement.

$\therefore$ y = 3x + 50° = 3(26°) + 50° = 128°.

Mini Revision Test # 01

DIRECTIONS: *Fill in the blanks.*

1. A part of a line with one end point is called a.....
2. Two adjacent angles are a linear pair of angles if and only if they are.......
3. A line intersecting two other lines is called a.....
4. Two angles are said to be supplementary if their sum is .....
5. Two angles are said to be complementary if their sum is .....
6. The angle formed by the two rays AB and AC, is denoted by the symbol .... or ....
7. When two lines intersect, four angles are formed. The angles opposite each other are called ....
8. Two angles are .... if they share the same vertex and a common side but no angle is inside another angle.
9. If a ray stands on a line, the sum of the two adjacent angles so formed is ....
10. Two lines in the same plane that are perpendicular to the same line are ...... to each other.

Notes / Rough Work

POINT TO REMEMBER

If two complementary angles contain a° and b°, then a° + b° = 90°.

Challenge Problems # 01

1. What is the number of distinct triangles with integral valued sides and perimeter as 14? ***(Q. code - 110805001)***

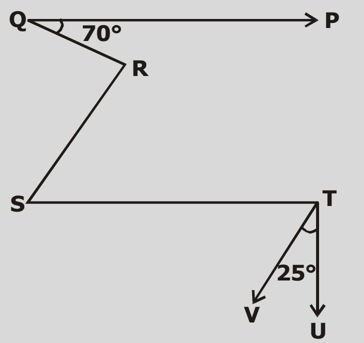
   (1) 6 (2) 5 (3) 4 (4) 3

2. The lengths of the sides of a triangle are in the ratio $1:\sqrt{2}:\sqrt{3}$. If the smallest angle is $\alpha°$, what are the measures of the other two angles? ***(Q. code - 110805002)***

   (1) $\sqrt{2}\alpha°, \sqrt{3}\alpha°$ (2) $(90 - \alpha)°, 90°$

   (3) $\frac{\alpha°}{2}, \frac{\alpha°}{3}$ (4) $2\alpha°, 3\alpha°$

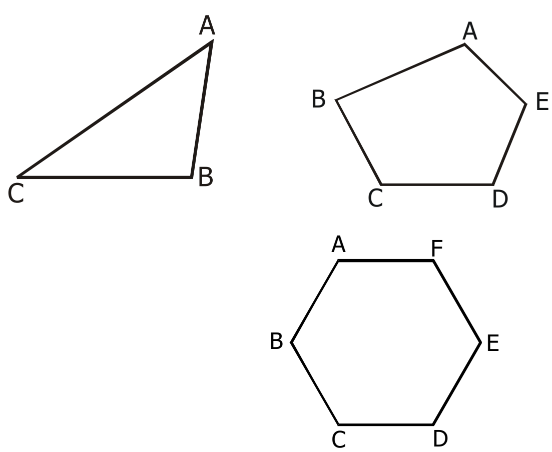
3. In the given figure, PQ || ST and TV || RS, TU $\perp$ ST. Find $\angle$QRS. ***(Q. code - 110805003)***



   (1) 120° (2) 125° (3) 135° (4) None of these

Polygon

Definition: A polygon is a closed figure bounded by straight lines. Each of the line-segments forming the polygon is called its side. The angle determined by two sides meeting at a vertex is called an angle of the polygon.



Remark: For n = 3, the polygon has the special name– triangle and similarly for n = 4, the polygon has the special name– quadrilateral.

Some other special names are

Pentagon (n = 5).

Hexagon (n = 6).

Notes / Rough Work

POINT TO REMEMBER

Adjacent angles are complementary if their exterior sides are perpendicular to each other.

Definition: A polygon P_1P_2.... P_n is called convex if for each side of the polygon, the line containing that side has all the other vertices on the same side of it.

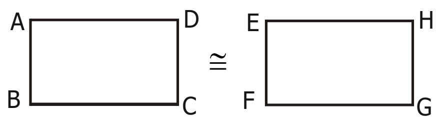
All the three figures given (triangle, pentagon and hexagon) are convex polygons.

Regular polygon : A polygon is called a regular polygon if all its sides and angles are equal.

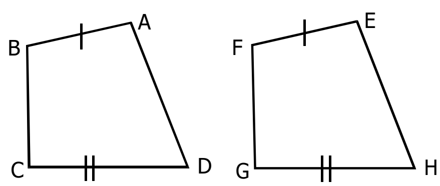
In the given figure, the hexagon is a regular polygon.

Congruent and similar polygons

If two polygons have equal corresponding angles and equal corresponding sides, they are said to be **congruent.** Congruent polygons have the same size and shape. The symbol for congruence is ≅.

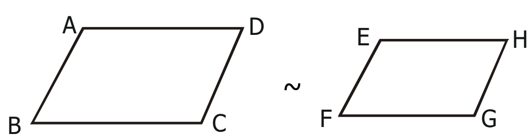


When two sides of the polygons are equal, we indicate the fact by drawing the same number of short lines through the equal sides.

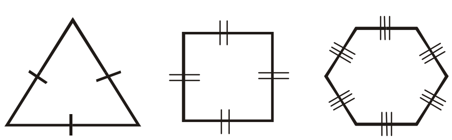


This indicates that AB = EF and CD = GH.

Two polygons with equal corresponding angles and corresponding sides in proportion are said to be **similar.** The symbol for similar is ~.



Similar figures have the same shape but not necessarily the same size.



Triangles and their types

A triangle is a polygon with three sides. Triangles are classified by measuring their sides and angles.

The sum of the angles of a plane triangle is always 180°. The symbol for a triangle is Δ. The sum of any two sides of a triangle is always greater than the third side.

Equilateral

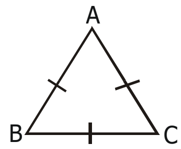
Notes / Rough Work

Equilateral triangles have equal sides and equal angles. Each angle measures 60°.

In the Δ ABC,

AB = AC = BC.

$\therefore \angle A = \angle B = \angle C = 60°$.



☞ Area $= \frac{\sqrt{3}}{4}a^2$.

Isosceles

Isosceles triangles have two sides equal. The angles opposite to the equal sides are equal. The two equal angles are sometimes called the base angles and the third angle is called the vertex angle. Note that an equilateral triangle is isosceles.

In the ΔFGH,

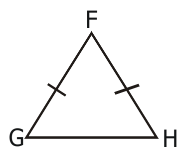
FG = FH.

FG $\neq$ GH.

$\angle G = \angle H$.

$\angle F$ is vertex angle.

$\angle G$ and $\angle H$ are base angles.



☞ Area $= \frac{b}{4}\sqrt{(4a^2 - b^2)}$.

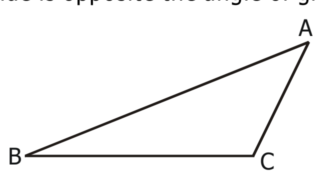
Scalene

Scalene triangles have all three sides of different length and all angles of different measure. In a scalene triangle, the shortest side is opposite the angle of smallest measure, and the longest side is opposite the angle of greatest measure.

In the ΔABC,

AB > BC > CA.

$\therefore \angle C > \angle A > \angle B$.



E7. Find the area of a triangle whose sides are 2.22 m, 2.46 m and 1.90 m.

Sol. If the three sides a, b, c, of a triangle are given, we know that the area of the triangle is obtained by the formula.

Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.

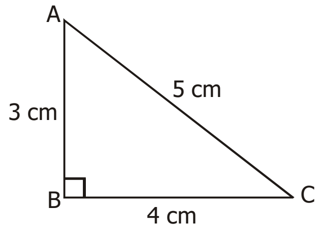
In this case,

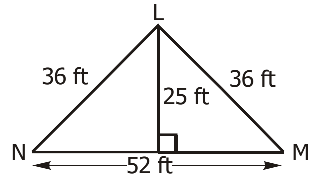
$s = \frac{2.22 + 2.46 + 1.90}{2} m = \frac{6.58}{2} m$ = 3.29 m = 329 cm.

$\therefore \quad A = \sqrt{329\ (329-222)(329-246)(329-190)}$ sq. cm.

$\Rightarrow$ Area = 20153 sq.cm = 2.015 sq.m.

E8. Find the perimeter and area of Δ ABC & Δ LMN.





Sol. Perimeter of Δ ABC = 3 + 4 + 5 = 12 cm.

Area of Δ ABC = $\frac{1}{2} \times 4 \times 3 = 6$ sq.cm.

Perimeter of Δ LMN = 36 + 36 + 52 = 124 ft.

Area of Δ LMN = $\frac{1}{2} \times 52 \times 25 = 650$ sq.ft.

E9. Find the altitude of a right triangle with area 50 sq. ft and base 8 ft.

Sol. Area of a right triangle = $\frac{1}{2} \times$ base $\times$ altitude.

$\Rightarrow 50 = \frac{1}{2} \times 8 \times$ altitude.

$\therefore$ altitude = 12.5 ft.

Right angled triangle

Right angled triangle contains one right angle. Since the right angle is 90°, the other two angles are complementary. They may or may not be equal to each other. The side of a right triangle opposite the right angle is called the hypotenuse. The other two sides are called legs. The **Pythagorean theorem** states that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

In the ΔABC,

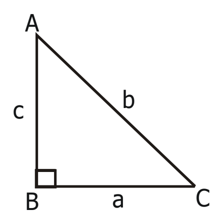
AC is the hypotenuse.

AB and BC are legs.

$\angle B = 90°$.

$\therefore \angle A + \angle C = 90°$.

$\Rightarrow a^2 + c^2 = b^2$.



E10. ABC is a right triangle with $\angle B = 90°$. If AB = 12 and BC = 5, then what is the length of AC?

Sol. $AB^2 + BC^2 = AC^2$.

$\Rightarrow AC^2 = 12^2 + 5^2 = 144 + 25 = 169$.

$\therefore$ AC = 13.

Notes / Rough Work

E11. Show that a triangle with sides 15, 8 and 17 is a right triangle.

Notes / Rough Work

Sol. The triangle will be a right triangle if $a^2 + b^2 = c^2$.(as shown above)

$\therefore 15^2 + 8^2 = 17^2$.

$\Rightarrow 225 + 64 = 289$.

$\therefore$ the triangle is a right triangle and 17 is the length of the hypotenuse.

Properties of triangles

- Sum of the three interior angles of a triangle is 180°.
- When one side is extended in any direction, an angle is formed with another side. This is called the exterior angle.
- Interior angle + corresponding exterior angle = 180°.
- An exterior angle = Sum of the other two interior angles not adjacent to it.
- Sum of any two sides is always greater than the third side.
- A triangle must have at least two acute angles.

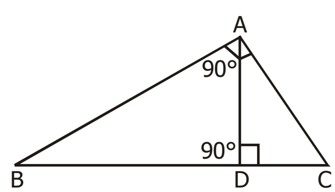
Properties of similar triangles

If two triangles are similar, then ratio of sides = ratio of heights = ratio of perimeters = ratio of medians = ratio of angle bisectors = ratio of inradii = ratio of circumradii.

In the above result, ratio of sides refers to the ratio of corresponding sides etc.

Also, ratio of areas = ratio of squares of corresponding sides.

Right triangle



ABC is a right triangle with $\angle A = 90°$.

If AD is perpendicular to BC, then

(a) Triangle ABD ~ Triangle CBA and $BA^2 = BC \times BD$.

(b) Triangle ACD ~ Triangle BCA and $CA^2 = CB \times CD$.

(c) Triangle ABD ~ Triangle CAD and $DA^2 = DB \times DC$.

Two triangles are similar

Two triangles are similar if the angles of one of the triangles are respectively equal to the angles of the other, and the corresponding sides are proportional.

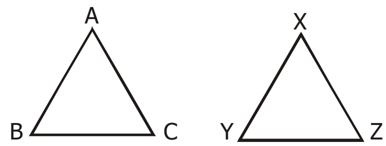
Notes / Rough Work

Theorems

(Equiangular triangles) A.A.A.

If two triangles are equiangular, their corresponding sides are proportional. In triangles ABC and XYZ,

if ∠A = ∠X, ∠B = ∠Y, ∠C = ∠Z, then AB/XY = AC/XZ = BC/YZ.



(3 sides proportional)

If 2 triangles have their corresponding sides proportional, then they are equiangular. In triangles ABC and XYZ,

if AB/XY = AC/XZ = BC/YZ, then ∠A =∠X, ∠B =∠Y, ∠C =∠Z.

(Ratio of two sides and included angle)

If one angle of the triangle is equal to one of the angles of the other triangle and the sides containing these angles are proportional, then the triangles are similar.

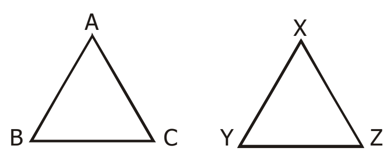
If ∠A = ∠X and AB/XY = AC/XZ,

then ∠B = ∠Y, ∠C = ∠Z.

If triangles ABC and XYZ are similar, then it is denoted by ΔABC ~ ΔXYZ.

Congruency of triangles

Two triangles are said to be **'CONGRUENT'** if they are equal in all respects (sides and angles).



The three sides of one of the triangle must be equal to the three sides of the other, respectively.

The three angles of the first triangle must be equal to the three angles of the other respectively. Thus, if Δ ABC and Δ XYZ are congruent, (represented as Δ ABC ≅ Δ XYZ), then

AB = XY, AC = XZ, BC = YZ and

∠A = ∠X, ∠B = ∠Y, ∠C = ∠Z.

Theorems

(Side . angle . side) S.A.S.

If AB = XY, ∠A = ∠X, AC = XZ, then triangle ABC ≅ triangle XYZ.

(Angle . angle . side) A.A.S.

If $\angle B = \angle Y$, $\angle C = \angle Z$, AC = XZ, then triangle ABC ≅ triangle XYZ.

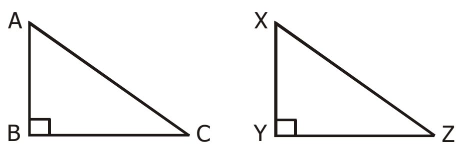
(Angle . side . angle) A.S.A.

If $\angle B = \angle Y$, BC = YZ, $\angle C = \angle Z$, then triangle ABC ≅ triangle XYZ.

(Side . side . side) S.S.S.

If AB = XY, AC = XZ, BC = YZ, triangle ABC ≅ triangle XYZ.

(Right angle hypotenuse side) (R.H.S)



If $\angle B = \angle Y = 90°$, Hypotenuses AC = XZ, AB = XY, then triangle ABC ≅ triangle XYZ.

Note: Congruent triangles are definitely similar but similar triangles may not be congruent.

Mini Revision Test # 02

DIRECTIONS: *Answer the questions.*

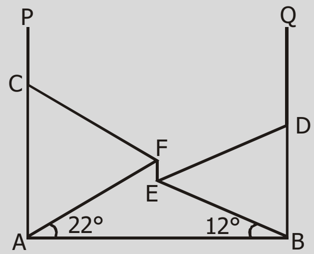
1. What is the measure of each acute angle of an isosceles right angled triangle?
2. One of the acute angles of a right triangle is 42°. Find the measure of the other acute angle.
3. Two angles of a triangle measure 92° and 58° respectively. Find the measure of the third angle.
4. One of the acute angles of a right triangle is double the other. Find the measure of its smallest angle.
5. Each of the equal angles of an isosceles triangle is double the third angle. What is the measure of the third angle?
6. The measures of the angles of a triangle are in the ratio 2 : 3 : 4. Find the measure of its greatest angle.
7. One of the angles of a triangle is equal to the sum of the other two. Find the measure of the greatest angle.
8. What is the measure of each angle of an equilateral triangle?
9. The measures of the acute angles of a right triangle are in the ratio 1 : 5. Find the measure of its angles.
10. Match the following :

i.	Right triangle	i.	120°, 28°, 32°.
ii.	Isosceles right triangle	ii.	60°, 60°, 60°.
iii.	Obtuse scalene triangle	iii.	90°, 42°, 48°.
iv.	Equilateral triangle	iv.	108°, 36°, 36°.
v.	Acute angled isosceles triangle	v.	45°, 45°, 90°.
vi.	Obtuse isosceles triangle	vi.	65°, 50°, 65°.

Notes / Rough Work

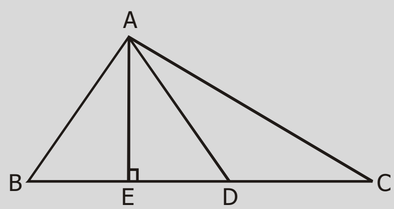
Challenge Problems # 02

1. In the given figure AP ⊥ AB, AP || EF || BQ and AF || DE. Find ∠DEB.
(Q. code - 110806001)



(1) 20° (2) 22° (3) 26° (4) 34°

2. In the given ΔABC (not drawn to scale), D is the midpoint of the side BC, such that ∠ADB is acute and seg AE ⊥ seg BC. If AE = DE = BE, then $\frac{AC}{BD}$ =?
(Q. code - 110806002)



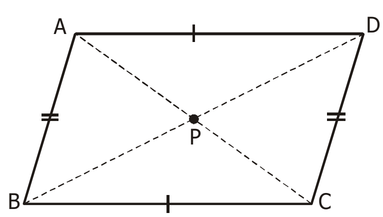
(1) 1 (2) $\sqrt{5}$ (3) $\sqrt{2.5}$ (4) $\frac{\sqrt{5}}{2}$

3. In triangle ABC, AB = 20 cm and BC = 30 cm and BD is an angle bisector (point D lies on AC). Point E is taken on BC so that DE is parallel to AB and point K is taken on DC so that EK is parallel to BD. If (AD - KC) = 1, then what is the length of AC? *(Q. code - 110806003)*
(1) 21 (2) 26 (3) 25 (4) None of these

Quadrilaterals

A quadrilateral is a polygon of four sides. The sum of the interior angles of a quadrilateral is 360°.

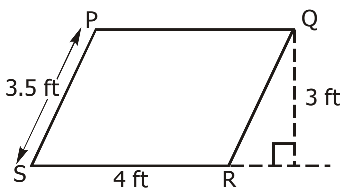
Parallelogram



AD || BC	∠A + ∠B = 180°
AD = BC	Δ ABD ≅ Δ CDB
AB || DC	Δ ABC ≅ Δ CDA
AB = DC	AP = PC
∠D = ∠B	BP = PD
∠A = ∠C	

Notes / Rough Work

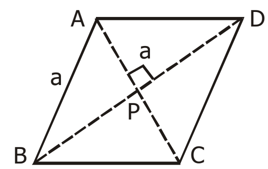
E12. Find the perimeter and the area of the parallelogram PQRS given below.



Sol. Perimeter = 2(3.5 + 4) = 15 ft.

Area = 4 × 3 = 12 sq.ft.

Rhombus

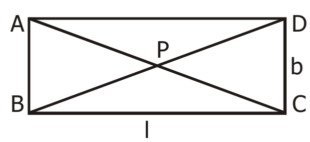


If in a parallelogram, all the sides are equal, then the parallelogram so formed is a rhombus.

Some important properties

- $\angle DAB = \angle DCB$ & $\angle ADC = \angle ABC$.
- AP = PC, DP = PB & $\angle APD = 90°$.
- $\angle ADP = \angle PDC$, $\angle ABD = \angle PBC$.
- $\angle ADC + \angle DCB = 180°$ & $\angle DAB + \angle ABC = 180°$.

Rectangle



Area of a rectangle is the product of two adjacent sides.

$\therefore$ Area = ab.

E13. The perimeter of a rectangle is 36 cm. If the length of one of the sides is 12 cm, then find its area.

Sol. Let the length of the rectangle = 12 and breadth = y.

$\therefore 2 \times 12 + 2y = 36$.

$24 + 2y = 36$ or $2y = 12$. $\therefore y = 6$ cm.

$\Rightarrow$ Area = xy = 72 sq.cm.

E14. The length of a rectangle is 10 cm and its perimeter is 30 cm. Find the area of the rectangle.

Sol. Perimeter = 30 cm , length = 10 cm, breadth = y.

$\therefore 2 \times 10 + 2y = 30$.

$\therefore 20 + 2y = 30$.

$\Rightarrow 2y = 10$. i.e. $y = 5$.

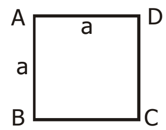
$\therefore$ area = $10 \times 5 = 50$ sq.cm.

Notes / Rough Work

Some important properties

- AD = BC, DC = AB, ∠A = ∠B = ∠C = ∠D = 90°.
- DP = PB = AP = PC.

Square



Area of a square = a^2.

Diagonal of a square = $a\sqrt{2}$.

Also, area of a square = $\frac{1}{2}(\text{diagonal})^2$.

E15. If the area of a square is 16 sq. ft, then find the length of each side.

Sol. Area of a square = $(\text{length of one side})^2$

$\therefore$ length of one side = $\sqrt{16}$ = 4 ft.

E16. If the perimeter of a square is 24 inches, then what is its area?

Sol. If perimeter is 24 inches, length of each side

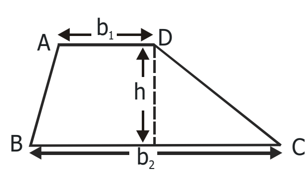
= 24/4 = 6 inches.

$\therefore$ Area = $(6)^2$ = 36 sq. inches.

Some important properties

- ∠A = ∠B = ∠C = ∠D = 90°.
- AB = BC = CD = DA.

Trapezium (Trapezoid)



AD || BC.

AD and BC are bases.

AB and DC are legs.

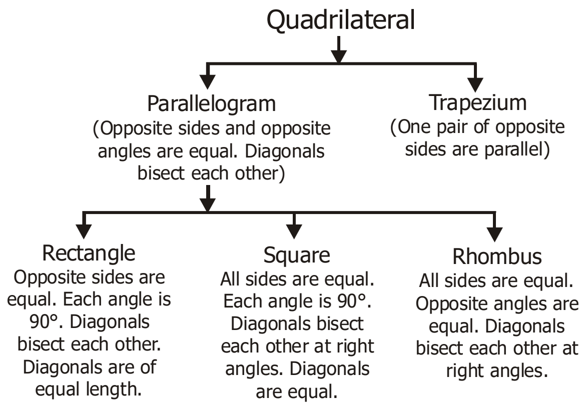
h = altitude.

Imp.

If a quadrilateral is not a parallelogram or a trapezoid but it is irregularly shaped, its area can be found by dividing it into triangles, attempting to find the area of each, and adding the results.

Notes / Rough Work

The chart below shows at a glance the properties of each type of quadrilateral.



Notes / Rough Work

☞ Important results and formulae

- The sum of all the angles of a triangle is 180°.
- The sum of any two sides of a triangle is greater than the third side.
- Exterior angle of a triangle is equal to the sum of opposite interior angles.
- Conditions of congruence of triangles are SAS, SSS, ASA and RHS.
- Angles opposite to equal sides in a triangle are equal and vice versa.
- If two sides of a triangle are unequal, the greater side has greater angle opposite to it and vice versa.
- If the square of the longest side of a triangle is equal to the sum of squares of the other two sides, the triangle is right-angled. Angle opposite to the longest side is a right angle.
- If the square of the longest side of a triangle is greater than the sum of squares of the other two sides, the triangle is obtuse-angled. Angle opposite to the longest side is obtuse.
- If the square of the longest side of a triangle is less than the sum of squares of the other two sides, the triangle is acute-angled.
- Perimeter of a triangle is greater than the sum of its medians.
- Triangles on the same or congruent bases and of the same altitude are equal in area.
- Triangles equal in area and on the same or congruent bases are of the same altitude.
- Parallelograms have opposite sides and opposite angles equal.
- Diagonal of a parallelogram divides it into two congruent triangles.
- Diagonals of parallelogram bisect each other. If a pair of opposite sides of a quadrilateral are parallel and equal, the quadrilateral is a parallelogram.
- The line–segment joining midpoints of two sides of a triangle is parallel to the third side and half of it. Conversely, the line drawn through the midpoint of one side of a triangle parallel to the other, passes through the midpoint of the third.
- Conditions of similarity of two triangles are SAS, SSS, ASA and AA.
- Areas of similar triangles are in the ratio of squares of corresponding sides, altitudes or medians.
- The diagonals of a rhombus bisect each other at right angles. The figure formed by joining the midpoints of the sides of a rhombus is a rectangle.
- The diagonals of a rectangle are equal and bisect each other.
- The diagonals of a square are equal and bisect each other at right angle.

Mini Revision Test # 03

DIRECTIONS: *Determine whether the given statements are TRUE or FALSE.*

1. One side of a rectangle is 4 m and its diagonal is 5 m. The perimeter of the rectangle is 12 m.
2. The diagonal of a square field measures 50 m. Its area is 1250 sq.m.
3. The area of an equilateral triangle with side 10 cm is $25\sqrt{5}$ sq.cm.
4. A rectangular plot is 110 m by 65 m. It has a path 2.5 m wide all round it on the inside. The area of the path is 750 sq.m.
5. The area of a triangle is 48 sq.cm. If its base is 12 cm, its altitude is 8 cm.

DIRECTIONS: *Fill in the blanks.*

6. An isosceles right triangle has area 200 sq.cm. The length of its hypotenuse is .....
7. The lengths of the diagonals of the rhombus are 18 cm and 16 cm. The area of the rhombus is .....
8. The area of an equilateral triangle whose side is 12 cm is ....
9. The area of a parallelogram whose base is 4 cm and the height is 5 cm is .....
10. The area of rectangle whose sides are in the ratio 3 : 2, is 2400 sq.cm. Its length is ......

Challenge Problems # 03

1. Neeraj has a rectangular field of size 20 × 40 sq. mt. He has to mow the field with a moving machine of width 1 mt. If he mows the field from the extremes to the centre, then the number of rounds taken by him to mow half of the field will be ***(Q. code - 110807001)***

   (1) 3.5 (2) 3.8 (3) 3 (4) 4

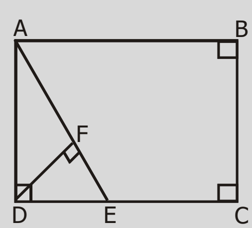
2. A square, whose side is 2 meters, has its corners cut away so as to form an octagon with all sides equal. Then the length of each side of the octagon, in meters is ***(Q. code - 110807002)***

   (1) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (2) $\frac{2}{\sqrt{2}+1}$ (3) $\frac{2}{\sqrt{2}-1}$ (4) $\frac{\sqrt{2}}{\sqrt{2}-1}$

3. Four villages lie at the vertices of a square of side 1 km. What is the smallest length of road needed to link all village together?
   (Q. code - 110807003)

   (1) 3 km (2) 2.73 km (3) 2.88 km (4) 4 km

4. In the above figure, DF = 4 units A(Δ DFE) = 6 sq. units, DE $= \frac{1}{2}$ EC, find the area of □ABCD.
   (Q. code - 110807004)

   (1) 100 sq. units
   (2) $90\sqrt{3}$ sq.units
   (3) $60\sqrt{3}$ sq. units
   (4) None of these

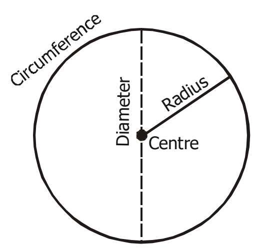
CIRCLES & MENSURATION

For online DCS, please enter the Q. code to view the solutions on the SRC.

Notes / Rough Work

Circle

The following figure shows the parts of a circle.



Circle

Centre of a circle is the point within that circle from which all the straight lines drawn to the circumference are equal in length.

Circumference of a circle is the length of the outer periphery of the circle. It is also known as the perimeter of the circle.

Radius of a circle is the distance from the centre of the circle to the circumference.

Chord of a circle is a straight line that divides the circle into two parts.

Diameter of a circle is the chord that passes through the centre of the circle. The diameter is twice the length of the radius .

In case of the diameter, it is the largest possible chord that can be drawn in a circle.

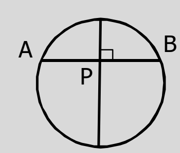
Imp.

The problem of finding lengths and areas related to figures with curved boundaries is of practical importance in our daily life.

POINT TO REMEMBER

A circle is symmetrical about every diameter. Hence any chord AB perpendicular to a diameter is bisected by the diameter.

Also, any chord bisected by a diameter is perpendicular to the diameter.



Notes / Rough Work

If A and C denote the area and perimeter (also called circumference) respectively, of a circle of radius r, then

$A = \pi r^2$, $C = 2\pi r$

Here π (a letter of Greek alphabet) is the ratio of circumference to the diameter of a circle. It stands for a particular irrational number whose value is given, approximately to two decimal places, by 3.14 or by the fraction $\frac{22}{7}$.

E1. Find the circumference of a circle with area 25π sq ft.

Sol. Area of the circle = $\pi r^2 = 25\pi$.

$\therefore$ r = 5 ft.

$\therefore$ circumference = $2\pi r = 10\pi$ ft.

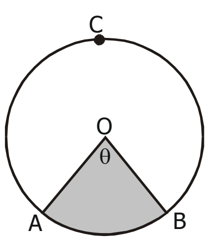
E2. Find the area of a circle with circumference 30π cm.

Sol. Circumference = $2\pi r = 30\pi$

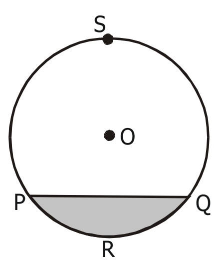
r = 15 cm.

$\therefore$ area = $\pi r^2 = \pi \times 15^2 = 225\pi$ sq. cm.

Sector and segment of a circle: See the figures given below.



(i)



(ii)

In figure (i) note that the area enclosed by a circle is divided into two regions, viz., OBA (shaded) and OBCA (non-shaded). These regions are called major and minor sectors respectively.

A minor sector has an angle θ, subtended at the centre of the circle. The boundary of a sector consists of arc of the circle and the two lines OA and OB.

In fig (ii), the area enclosed by a circle is divided into two segments. Here segment PRQ (shaded) is the minor one and segment PQS (non-shaded) is the major one. The boundary of a segment consists of an arc of the circle and the chord dividing the circle into segments.

Area of a sector and length of arc

The arc length l and the area A of a sector of angle θ in a circle of radius r, are given by

$l = \frac{\pi r\theta}{180}$, $A = \frac{\pi r^2\theta}{360}$. We also note that $A = \frac{1}{2}lr$.

E3. Complete the following table.

Radius r	r^2	Area A	Circumference C
1	1	--	--
2	4	--	--
3	9	--	--
4	16	--	--

Sol.

Radius r	r^2	Area = πr^2	C = $2\pi r$
1	1	π	2π
2	4	4π	4π
3	9	9π	6π
4	16	16π	8π

E4. The length of the minute hand of a clock is 7 cm. Find the area swept by the minute hand in 10 minutes. (Take $\pi = 22/7$).

Sol. The minute hand describes a circle of radius 7 cm.

It describes an angle of $\frac{360°}{60} = 6°$ per minute.

Thus, we get a sector of angle 6° and radius 7 cm per minute.

Its area $\frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times \frac{7 \times 7 \times 6}{360} = \frac{77}{30}$ = 2.567 sq. cm.

$\therefore$ Area of the sector described in one minute = 2.567 sq. cm.

$\Rightarrow$ Area described in 10 minutes = 2.567 × 10 sq. cm = 25.67 sq. cm.

E5. A sector is cut from a circle of radius 21 cm. The angle of the sector is 150°. Find the length of its arc and area.

Sol. Length of the arc of the sector,

$l = \frac{150}{180} \times \pi r$ cm.

$= \frac{150}{180} \times \frac{22}{7} \times 21$ cm = 55 cm.

Area of the sector $= \frac{150}{360} \times \pi r^2$ sq. cm.

$= \frac{150}{360} \times \frac{22}{7} \times 21 \times 21\ cm^2$ = 577.5 sq. cm.

Notes / Rough Work

Mini Revision Test # 01

DIRECTIONS: *Determine whether the given statements are TRUE or FALSE.*

1. The sum of the interior angles of a pentagon is 540°.
2. The perimeter of a regular hexagon with side a is 6a.
3. The circumference of a circle whose radius is 14 cm is 176 cm.
4. The perimeter of a regular hexagon with side 8 cm is 48 cm.
5. The area of a quadrant of a circle with radius 14 cm is 148 sq. cm.

DIRECTIONS: *Fill in the blanks.*

6. All circles measure .... degrees.
7. The area of a regular hexagon with side 8 cm is .....
8. If the radius of a circle is decreased by 50%, its area will decrease by ....
9. If the diameter of a circle is increased by 100%, its area is increased by ....
10. The length of minute hand on a wall clock is 14 cm. The area swept by it in 30 minutes is ....

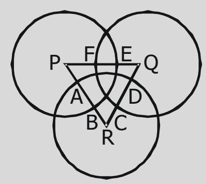
Challenge Problems # 01

1. The length of the common chord of two circles of radii 15 cm and 20 cm whose centers are 25 cm apart, is (in cm) ***(Q. code - 110905001)***

   (1) 24 (2) 25 (3) 15 (4) 20

2. Given below are three circles, each of radius 20 and centres at P, Q and R; further AB = 5, CD = 10 and EF =12. What is the perimeter of the triangle PQR?

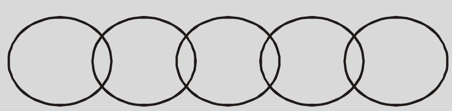
   (Q. code - 110905002)



   (1) 120 (2) 66
   (3) 93 (4) 87

3. There is a square of side x and two circle are drawn with radius x and from opposite vertices Find the area enclosed? ***(Q. code - 110905003)***

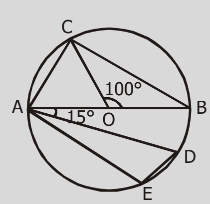
4. There are 5 congruent circles arranged in a row such that their centers are in the same straight line and the distance between their centers is less than their diameter, as shown. If a circle from either of the ends is removed, the area of the entire ensemble reduces by 22 square cm and if the circle in the middle is removed, the area of the entire ensemble reduces by 20 square cms. The area of the ensemble is (square cm) ***(Q. code - 110905004)***



   (1) 100 (2) 104 (3) 108 (4) 112

5. In the given figure, O is the center of the circle. BOC = 100 degrees and BAD = 15 degrees. If AE || CB, what is angle ADE (degrees)?

   (Q. code - 110905005)



   (1) 30 (2) 35
   (3) 45 (4) 50

Notes / Rough Work

POINT TO REMEMBER

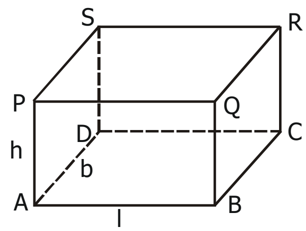
In equal circles or in the same circle, equal chords are equidistant from the centre. Chords, which are equidistant from the centre, are equal.

Cube and cuboid

Notes / Rough Work

A rectangular brick is an example of a cuboid. A chalk box is another example. A cuboid is also called a rectangular parallelopiped.

It has six faces, each one a rectangle. All faces intersect at right angles. Opposite faces are parallel and congruent. There are three pairs of parallel faces. Two adjacent faces meet along a line segment called an edge. A cuboid has twelve edges.



The faces are the rectangles ABCD, PQRS, APQB, QBCR, CRSD and DSPA. The sum of the areas of these six faces is the total surface area of the cuboid. Rectangles ABCD, PQRS form a pair of opposite faces. Its eight vertices are A, B, C, D, P, Q, R and S.

It is easily seen from the figure that

AB = PQ = RS = DC = l, say.

Similarly BC = AD = PS = QR = b.

and AP = BQ = CR = DS = h.

We may say that l is the length, b, the breadth and h, the height of the cuboid.

When $l = b = h \neq 0$, i.e., when all the sides of a cuboid are equal in length, it is called a cube.

volume of cuboid = $l \times b \times h$.

Total surface area of cuboid = $2[lb + bh + hl]$.

Volume of cube of side a = a^3.

Total surface area of cube of side a = $6a^2$.

E6. The measurements of a cuboidal luggage box are 48 cm, 36 cm and 28 cm. Find the volume of the box. How many sq.cm of cloth is required to make a cover for the box?

Sol. (i) The required volume

$= 48 \times 36 \times 28 = 48384$ cu.cm.

(ii) The quantity of cloth required to make the cover of the box = Total surface area of the box

$= 2[48 \times 36 + 36 \times 28 + 28 \times 48]$sq.cm.

$= 2 \times 48 \times [1 \times 36 + 3 \times 7 + 28 \times 1]$sq.cm.

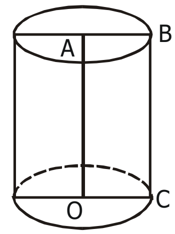
$= 96 \times 85$ sq.cm = 8160 sq.cm.

POINT TO REMEMBER

An angle at the centre of a circle is twice any angle at the circumference subtended by the same arc.

Cylinder

An object, having lateral (curved) surface and congruent circular cross section, is called a circular cylinder. The line joining the centres of these circular cross sections is called the axis of the cylinder. If the axis of the cylinder is perpendicular to the circular sections, then the cylinder is called a right circular cylinder otherwise it is called an oblique cylinder.



Hence, the area of the curved surface = $2\pi rh$.

The space enclosed by a cylinder as given above has a volume.

Total surface area = $2\pi r^2 + 2\pi rh$.

The volume of a cylinder with radius r and height h

= area of the base multiplied by the height = $\pi r^2 h$.

E7. The diameter of a cylinder is 28 cm and its height is 20 cm. Find (i) the curved surface area, (ii) the total surface area and (iii) the volume of the cylinder. (Assume $\pi = \frac{22}{7}$).

Sol. (i) Curved surface area = $2\pi rh$.

$= 2 \times \frac{22}{7} \times 14 \times 20 \text{ cm}^2$ = 1760 sq.cm.

(ii) Total surface area = $2\pi r(h + r)$.

$= 2 \times \frac{22}{7} \times 14 \times (20 + 14) \text{ cm}^2$ = 88 × 34 = 2992 sq.cm.

(iii) Volume = $\pi r^2 h$.

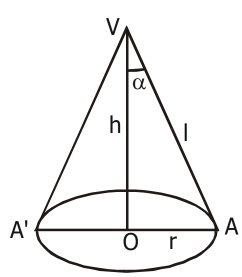
$= \frac{22}{7} \times 14 \times 14 \times 20 \text{ cm}^3$ = 12320 cu.cm.

$\therefore$ Capacity of the cylinder = 12.32 litres.

(1000 cu.cm = 1 litre).

Cone

A cone has a curved surface with a vertex and a circular base. In the figure V is the vertex, base is a circle with centre O and radius OA = r.



We deal with a right circular cone, in which the line VO (height) is perpendicular to the base. Hereafter we will mean by cone, a right circular cone.

The distance of the vertex from any point on the circular base is called the slant height of the cone and is usually denoted by l.

Notes / Rough Work

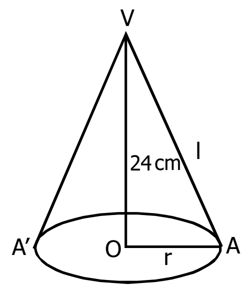
A cone of radius r, height h has slant height $l = \sqrt{r^2 + h^2}$.

Volume = $\frac{1}{3}\pi r^2 h$.

Curved surface area = πrl.

Total surface area = $\pi r(l + r)$.

E8. A cone of height 24 cm has curved surface area 550 sq.cm. Find its volume. (Assume $\pi = \frac{22}{7}$).



Sol. If r cm is the radius of the base and l cm the slant height (see figure), we know that

OA = r, OV = h, AV = l,

$\therefore l^2 = r^2 + 24^2 = r^2 + 576$.

$\Rightarrow \pi rl = 550 \Rightarrow rl = 550 \times \frac{7}{22}$.

Squaring both sides, we get $r^2 l^2 = (25 \times 7)^2$.

$\Rightarrow r^2 (r^2 + 576) = (25 \times 7)^2 = 625 \times 49$

or $r^4 + 576r^2 - (625 \times 49) = 0$

or $(r^2 - 49)(r^2 + 625) = 0$

Here $r^2 + 625 \neq 0$. $\therefore r^2 - 49 = 0$.

$\therefore$ radius of the base (r) = 7 cm.

$\Rightarrow$ Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$ cu. cm. = 1232 cu.cm.

E9. A sector of a circle of radius 12 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. Find the volume of the cone. (Take $\pi = \frac{22}{7}$)

Sol. The length of the arc of the sector $= \frac{120}{180} \times \pi \times 12$ cm $= 8\pi$ cm.

$\therefore$ perimeter of the base of the cone = 8π cm.

If r cm is the radius of the base, then $2\pi r = 8\pi$.

$\Rightarrow$ radius r = 4 cm.

The slant height of the cone (l) = 12 cm.

Height of the cone (h) $= \sqrt{l^2 - r^2}$ cm.

$= \sqrt{144 - 16}$ cm $= \sqrt{128}$ cm $= 8\sqrt{2}$ cm.

Let V be the volume of the cone. Then

$V = \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 8\sqrt{2}$ = 189.5 cu.cm· (approximately).

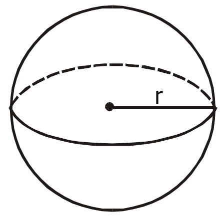
POINT TO REMEMBER

Every angle at the circumference subtended by the diameter of a circle is a right angle triangle.

Sphere: surface area and volume

A hollow sphere can be described as a set of all points in space (of three dimensions) equidistant from a fixed point, called the centre. Tennis ball and fully blown volley ball are two examples of hollow spheres.

The space enclosed by a sphere has a volume. It is customary to call this volume as the volume of the sphere.



A sphere of radius r has, volume $= \frac{4}{3}\pi r^3$.
Among all solids of given volume, the sphere has smallest surface area. Surface area of sphere = $4\pi r^2$.

Imp.

When a plane through the centre of a sphere divides the sphere into two equal parts, each part is called a hemisphere.

A hemisphere of radius r has

Volume $= \frac{2}{3}\pi r^3$, Curved surface area = $2\pi r^2$ and

Total surface area = $2\pi r^2 + \pi r^2 = 3\pi r^2$.

E10. The surface area of a sphere is 5544 sq.m. Find its volume.

Sol. We have $4\pi r^2 = 5544$.

$\therefore\ r^2 = \frac{5544}{4\pi} = 441$. $\quad \therefore$ r = 21 i.e., radius = 21 m.

Now, Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$ = 38808 cu.m.

Mini Revision Test # 02

DIRECTIONS: *Answer the following questions.*

1. What is the volume of a cuboid with dimensions 12 m, 10 m and 8 m?
2. What is the total surface area of a cube with side 5 cm?
3. Two cubes each with 12 cm edge are joined end to end. Find the volume of the resulting cuboid.
4. If the edge of a cube is increased by 50%, then find the percentage increase in its surface area.
5. A cylinder has a diameter of 14 m and its volume is 110 cu.m. Find the height of the cylinder.
6. What is the slant height of a cone with base radius 21 cm and height 28 cm?
7. How much canvas will it take to make conical tent 6 m in height and 8 m in diameter at the base?
8. What is the number of balls of diameter 2 cm each that can be made from a sphere of diameter 20 cm?
9. The surface area of a sphere is 616 sq.cm. Find its radius.
10. The surface area of a cube is 1944 sq.cm. Find its volume.

Challenge Problems # 02

Notes / Rough Work

POINT TO REMEMBER

For the same height and radius volume of cone is one-third that of the cylinder.

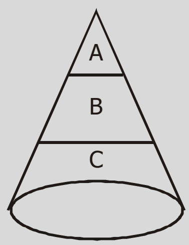
1. X wants to paint his room (four walls and the ceiling) which is cuboidal in shape. He wants to find out the cost of paint but he does not know the exact dimensions of his room. The only thing he knows is that the length, width and height of his room are in the ratio 5 : 3 : 4. Y, X's best friend, has a room which is 10% longer, 20% wider and 15% shorter (in height) than X's room and that his cost of paint was 4838.37. If the price of paint to be used by X is the same as that used by Y, what would be X's approximate cost of paint for painting his room (Rs.)? ***(Q. code - 110906001)***

(1) 4000 (2) 4200 (3) 4500 (4) 4100

2. A right circular cone of height h is cut by a plane parallel to the base and at a distance $\frac{h}{3}$ from the base, then the volumes of the resulting cone and the frustum are in the ratio ***(Q. code - 110906002)***

(1) 1 : 3 (2) 8 : 19 (3) 1 : 4 (4) 1 : 7

3. What is the ratio of volume of parts A : B : C if height of part A = height of part B = height of part C in the following figure? ***(Q. code - 110906003)***



4. A cone of radius 'r' is there whose slant height = diameter of the cone. Inside this cone, the largest possible sphere is placed. Inside this sphere, largest possible cube is placed. Insider this cube, largest possible cylinder is placed. What is the volume of the cylinder? ***(Q. code - 110906004)***

5. Three cylinders each of ht 16 cm and radius of base 4cm are placed on a plane so that each cylinder touches the other two. Then the volume of the region enclosed between the 3 cylinder is ***(Q. code - 110906005)***

(1) $90\,(4\sqrt{3}-\pi)$ (2) $98\,(2\sqrt{3}-\pi)$

(3) $98\,(\sqrt{3}-\pi)$ (4) $128\,(2\sqrt{3}-\pi)$

Notes / Rough Work

POINT TO REMEMBER

The cylinder is a prism, whereas cone is a pyramid.